Department of Mathematics

Pattamundai College, Pattamundai

3rd Semester

Theory of Real Functions (Analysis-II)

Core – 5

Sec–A

(Unit–1)

1. What is post Office function.
2. What is deleted neighbourhood
3. What is limit point of a set
4. What is the contact point of the set.
5. Write $\in$-δ definition of limit.
6. Use sequential limit from to obtain the limit of $f(x) = x^3+x^2-5$ at x=a.
7. Consider the greatest integer function $f(x) = [x]$. Show that $\lim_{x\to a} f(x)$ is not exist at any integral point.
8. If f is continuous then |f| is continuous. Justify your answer.
9. Show that the function $f(x) = 2x^2+3x+2$ is bouded in the interval [0,1].
10. Show that $f(x) = |x|$ is continious on IR but it is un bounded above.
11. Show that the function f: R→R defind by $f(x) = \frac{1}{1+x^2}$ is continous on R but the range set is neither open nor closed.
12. If c(x) = { f:x→R, f is continuous on x}, then c(x) is a vector space?
13. Write the necessary condition when two continouus function is summable.
14. Give an example of removable discontinuity.
15. Give an example of jump discontinuity.
16. Draw the graph of $f(x) = \frac{1}{x}$, which type of discontinuity gives at x =0?

(Unit–2)

17. Show that $f(x) = x^2$ is monotonic increasing function.
18. Write the difference between the monotonic decreasng and strictly decreasing function.
19. Show that a function is both increasing and decreasing then the function most be a constant.
20. A monotonic function can have a countable number of discontinuities of only the first kind. (T/F)
21. Write the definition of $\lim_{x\to a} f(n) = \infty$, by ($\in$–$\delta$) method.
22. Show that $\lim_{x\to 0} \frac{1}{|x|} = \infty$

P.T.O.

23. Show thath f and gh are two uniform conthinuos then show that f.g is not a uniform continuous.

24. Show that f(x) = x is uniform continous on R but f(x) =x^2 is not uniform continuous on R.

25. Give an example of function which is continous but not uniforhmly continuous.

26. Write the statement of Banach's contraction Principle.

27. Show that the set of differentiable function from a vector space.

28. Find the set of points where the following functions are not differentiable

i) f(x) = |s in x| (ii) f(x) = |x| + |x –1|

29. Let $f(x) = \begin{cases} x, & x \in Q \\ o, & x \in R-Q \end{cases}$ $f(x) = \begin{cases} x, & x \in Q \\ o, & x \in R-Q \end{cases}$

Show that f is diffentiable only at x = 0

(Unit–3)

30. Write two drawback of Rolle's Theorem.

31. Show that any function f(x) has a real root in (a,b) if f(a). f(b) < 0.

32. Examine the function $f(x) = \frac{x-1}{x}$, a =1, b =3 satisfy the mean value condition.

33. If f: R→R satiesfies $|f(x)- f(y) \leq k(x-y)^2$ for some constant k>0 and for all x,y∈R then f most be a constant function.

34. Examine the function f(x) = |x| satisfy the Rolle's conditions find the critical point it exist.

35. Show that $\lim_{x\to\infty} \frac{\log x}{x} = 0$ use L. Hospital rule.

36. Show that Cauchy's mean value theorem convert to Lagrange mean value theorem if we take g(x) =x.

37. Find the maximum value of f(x) = sin x+x in R.

38. Verify Cauchy's mean value theorem for the functions x^2 and x^3 in the interval [1,2]

(Unit–4)

39. Write the Taylor's series with schlomilch form of reminder.

40. Write the Taylor's series with Roche's form of remainder.

41. Write the Taylor's series with Cauchy's form of remainder.

42. Write the maclaurin's series of the function $f(x) = e^x$.

43. Write the maclaurins' series of the function f(x) = tan x.

44. Write the Maclaurin's series of the function f(x) = cos x.

45. Discuss the application of Lagrange's mean value treorem to $f(x) = \frac{1}{x}$ in [–1,1]

46. Write the statement of Generalised mean value theorem.

47. Write only the statement of Taylor's Theorem.

48. Let $f: I \to R$ be twice continuosly differentiable on I. Then f has a local minimum at c if $f''(c) > 0$. Prove it.

49. Find the 3rd derivative of $f(x) = \frac{\log x}{x}$

50. Write the difference between the Taylor's series and Maclaurin's series.

Sec–B

(Unit–1)

1. The function $f: R - \{0\} \to [-1,1]$ defined by $f(x) = \sin(\frac{1}{x})$ does not posses a limit as $x \to 0$.

2. If f has a limit, then it is unique.

3. Let $\lim_{x\to a} f(x)$, $\lim_{x\to a} g(x)$ exist, then show that $\lim_{x\to a} (f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.

4. Let $f(x) = \begin{cases} 1, & x \in Q \text{ and} \\ 1, & x \in Q^c \text{ and} \end{cases}$ $g(x) = \begin{cases} 0, & x \in Q \\ 1, & x \in Q^c \end{cases}$

   Then show that $\lim_{x\to a} \{f(x) + g(x)\}$ exist but $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ does not exist.

5. Prove that $\lim_{x\to 0} \frac{\sin x}{x} = 1$

6. If f be continuous at x_0 and let g be continuous at $f(x_0)$. Then the composition function gof in continous of x_0.

7. Let and g be real function defined over the same domain and are continous at xo. Then prove that f.g is also continous at xo.

8. Let $f: x \to R$ be continous and $a \in R$. Then $\{x \in x : f(x) > a\}$ is an open sub set of x: $\{x \in x : f(x) \geq a\}$ is a closed subset of x.

9. If f is continous on a closed bounded interval [a,b], f takes every value between any two of its value, the image of f is also a closed bounded interval.

10. Show that $\lim_{x\to\infty} (1+\frac{1}{x})^x = e$

11. Show that $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$.

12. Given an example of function f and g which are discontinuos such that f+g is continuous
13. Given an example a function which is continous but unbounded.
14. Let a function f satisfy, for all x, $y \in R$, $f(x+y) = f(x).\, f(y)$. If f is continuous at a, show that it is continous on R.
15. Prove that x = cosx for some $x \in (0, \frac{\pi}{2})$.

(Unit–2)

16. A function which is differentiable at a point is continuous at the point.
17. Given an example of a function which is continuos but not differentiable at that point.
18. Prove that $f(x) = x^{\frac{1}{3}}$ is continous on R but is not differentiable at 0.
19. Show that $\rho\,(e^{\alpha x}) = \alpha e^{\alpha x}\ \alpha \in R$. by the limit method.
20. Let $f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$. Show that f is not differentiable at x = 0
21. Write the mean value conditions.
22. Prove that every uniformly continuous function is continuous.
23. Suppose f(x) and g(x) are two uniformly continous then disprove by a numerical example. f(x).g(x) is not uniformly continuous.
24. Given an example of a function which is uniformly continous on [a,b] but not nuiformly continuous on R.
25. Show that a function f is satisfied the the conditioin $\frac{\partial f}{\partial x}$ is bounded on [a,b] then f(x) is uniformly continuous but the converse need not true.
26. Show that the sum and product of two differentiable function is differentiable.
27. Show that $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ the Left hand and Right derivative is exist but not equal.
28. Show that $f(x) \begin{cases} \frac{\sin 3x}{x} & x \neq 0 \\ 4 & x = 0 \end{cases}$ show that f(x) has removable discontinouity.
29. Given an example of 2nd kind discontinuity.
30. Show that $\lim_{x \to 0} x^2 \sin(\frac{1}{x}) = 0$

(Unit–3)

31. State and prove Generalised mean value theorem.
32. Show that the Cauchy's mean value theorem is a particular case of Lagramses mean value theorem.
33. Let f be differentiable on (a,b) then of $f'(x) \geq 0\ \forall x \in (a,b)$, f is montonically increasing.
34. Show that $10x^4 - 6x + 1 = 0$ has a root between 0 and 1.

35. Find the local manimum and minimum of $f(x) = 8x^5 - 15x^4 + 10x^2$

36. Examine of the following function satisfy the conditions of Rolle's theorem. $f(x) = x(x-a)^m (x-b)^m$, m.n∈NI on [a,b].

37. Show that $|\sin y - \sin x| \leq |y-x|$ in the interval [x,y].

38. Suppose that $co + \frac{c1}{2} + ... + \frac{cn}{n} + 1 = 0$ where $C_k \in IR$. Show that the equation $c_0 + c_1 x + ... + c_n x^n = 0$ has at least one root in the interval [0,1].

39. Show that $\lim_{x \to \infty} x \sin \frac{1}{x} = 1$.

40. Show that $\lim_{x \to 0} x^x = 1$

41. State and prove the Layrange meom value theorem.

42. Write the difference between the local max and global maximum.

43. Show that Taylor's theorem is a generalisation of Lagranse. mean valu theorem.

44. Suppose that f is twice differentiable in an open interval I and that $f''(x) = 0 \ \forall n \in I$. Show that $f(x) = ax+b$ for constant a and b.

45. Obtain the following inequality by use of mean value theorem. $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$ $x>0$.

(Unit – 4)

46. Find the n_{th} Taylor Polynomial of the function e^x at x =2.

47. Find a polynomial f(x) of degree 2 which satisfies $f(1) = 2$, $f'(1) = -1$ and $f''(1) = 2$.

48. Find the maximum and minimum of $f(x) = x^3 - 5x^4 + 5x^3 - 1$.

49. Show that the triangle of maximum area that can be inscribed in a circle is equilateral.

50. Find $f^n(x)$, for $n \in N$, $x \in R$ of f(x) = cosax.

51. Let $f(x) = |x^3|$ for $x \in R$. Find $f'(x)$, $f''(x)$, $f'''(x)$, $x \neq 0$ show that $f'''(0)$ does not exist.

(Sec – C)

Unit –1

1. Show that $\lim_{x \to a} \frac{x^3 - a^3}{x - a} = 3a^2$ by $(\in - \delta)$ method.

2. Let f: $x \to R$, $a \in R$. Then $\lim_{x \to a} f(x) = L$ if and only if $\lim_{n \to \infty} f(xn) = L$, for every sequence (xn) in x such that $x_n \neq a$ for n =1,2,3 ... and $\lim_{x \to \infty} xn = a$,

3. State abd prove Cauchy criterian of limit of function.

Or

Let f: X→R, a∈R. Then $\lim_{x\to a}$ f(x) exists if and only if for ∈>0. we can find δ>0 such that |f(x)–f(y)I <∈ where x,y∈N*(a,δ).

4. Let f: X→R. The following conditions are equivalent

a) f is continuous on x.

b) If O is an open subset of R then $f^{-1}(0)$ is an open subset of x.

c) If S is a closed subset of R, then $f^{-1}(S)$ is a closed subset of x.

5. Let f: [a,b]→R be continuous and set α be any real number between $\sup_{x\in[a,b]}$ f(x) and $\operatorname{Inf}_{x\in[a,b]}$ f(x). Then there exists a point c ∈ [a,b] such that f(c) =α.

Unit –2

6. State and prove Caratheodory's theorem.

7. Let f: (0,∞)→ R be defined by f(x) = $\frac{1}{x}$. Then f is uniformly continuous an [a,∞], a>0 but not uniformly continuous on (0,∞) though it is continous there.

8. Let S be a closed and bounded sub set of R. of f : S→R is continous, it is uniformly continuous on S.

9. Discuss the different kinds of discontinuity with numberical examples.

10. Discuss the continuity of f(x) = $\begin{cases} 1+x & -\infty < x < 0 \\ 1+[x]+\sin x, & 0 \le x < \frac{\pi}{2} \\ 3 & x \ge \frac{\pi}{2} \end{cases}$

Unit – 3

11. Let f: (a,b) → R satisry the following conditions

i) f assumes its local mamimum or minimum of x_0∈(a,b)

ii) f is diffentiable at x_0. Then f'(x_0) = 0, prove it.

12. If a and b are two roots of the equation f(x)=0 then there exists at least one root c of the equation f′(x) =0 such that a<c<b.

13. Let f and g be n times differentiable at a. Then $D^n(fg)(a) = \sum_{k=0}^{n}$. nck $(D^{n-k} f(a).\ (D^k g(aj)$.

14. State and prove Taylor's Theorem with Lagrange's remainder.

15. Show that the largest rectangle inscribed in a circle is a square.

Unit – 4

16. Show that Maclaurin's series of $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x4}{4} + \ldots$ for $|x| < 1$. Also find R_n, Where Rn = Lagrange's form.

17. Prove that a conical tent of a given capacity will require that least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

18. Show that $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$, find Rn Lagrange from.

19. Show that $e^{\pi} > \pi^{e}$, using Taylor's Theorem.

20. If f″ is continous on [a–δ, a+δ] for some δ>0. Show that $\lim_{h\to 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a)$.

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